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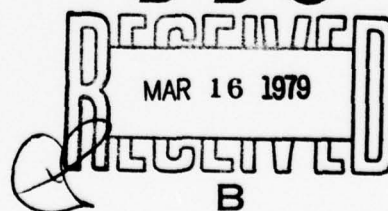
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Research Report CCS 319

A LINKAGE MODEL OF CAREER ASPIRATIONS AND PROMOTION OPPORTUNITIES OF STAFF[+]

by

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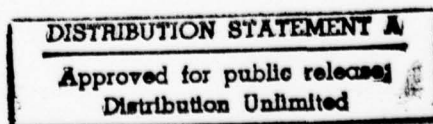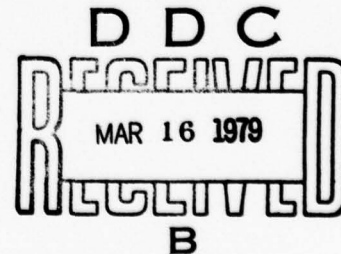
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ABSTRACT

This paper develops an organizational promotion model, based on a principle of equal employment opportunity, to effect linkages between promotion opportunities and organizational responses to legal compliance. The model considers individual attributes and the manner in which they are "weighted" as contributions to human resource valuation, and, hence, to the probabilities of promotion within a given career ladder. The connection of such human resource contributions to equal employment opportunity issues provides an insight into the development of a management tool for the discretionary weighting of promotion factors for future policy-making.

CONTENTS

1. Introduction

With the emergence of equal employment opportunity (EEO) considerations in the personnel management realm, complications have been introduced to manpower management systems. These complications are related to the design of procedures pertaining to recruitment, training, promotion and career development, and to the recalibration of the manpower system (i.e., the organizational-social structure) in order to comply with EEO law. Accountability of this new system must extend beyond the operational limits of goal attainment (i.e., monitoring the system to fulfill its prescribed responses) to provide a firm base or set of operational guidelines for personnel policy and planning. These must be realted to specific equal employment opportunity and affirmative action provisions that are legally defensible proof of compliance.

In 1964, the Civil Rights Act was passed, establishing what later became the Equal Employment Opportunity Commission. Title VII of the Act places broad prohibitions on employment discrimination on the basis of race, sex, or national origin. In 1967, the Department of Justice put a high priority on the enforcement of Title VII, with the specific objective of forcing case decisions on the law to provide Federal agencies with specific implications of the law. Such cases addressed the meaning of Title VII between 1968 and 1971, resulting in two interpretations. The narrower one saw Title VII as prohibiting only those employment acts which were consciously motivated by social prejudice. The broader interpretation, however,

was that Title VII prohibited any kind of employment practice that had a discriminatory impact, unless that practice was required by some business need. This second interpretation was upheld by the Supreme Court in 1971, stating that a test or requirement which has a discriminatory impact on a group that was previously the object of discrimination, is unlawful, unless the practice is required by a business necessity, (i.e., is job-related).[1] This ruling has largely defined the basic underpinnings of equal employment opportunity.

Meanwhile, in 1969, Executive Order 11246 was established, further strengthening the move toward EEO by providing for the design and implementation of affirmative action programs. Affirmative action was to supplement the equalizing effects of the job-related considerations of Title VII, by compensating for past imbalances in the manpower system via the imposition of some systematically determined high priority on minority mobility in the workforce.

In the present paper we develop a model that describes the interrelationship between EEO considerations and affirmative action policies, with allowances for separate consideration of the two. Couched in a framework of promotion opportunities for work force members, this model explores a multi-attribute approach to determining promotion probabilities as a function of (1) individual attribute combinations; (2) EEO law requiring like contributions to promotion opportunities by job-related individual attributes regardless of race, sex or national origin; and (3) affirmative action policy effects.

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[1]See Griggs vs. Duke Power Company. $\underline{/}\,6\,\underline{/}$.

The analysis of these promotion probabilities provides for a possible linkage to related works on manpower planning models of the kind developed by Charnes, Cooper, Lewis and Niehaus $\underline{/}$ 2 $\underline{/}$ and Stewman and Schinnar $\underline{/}$ 10 $\underline{/}$.[2] Furthermore, output from this work, in the form of promotion probabilities, provides a source of input required to run each of these other systems.

The organization of this paper is as follows. In section 2 we develop a conceptual backdrop for separating the effects of EEO and affirmative action policies. Implications of equal employment opportunity and affirmative action are detailed in sections 3 and 5, respectively, and their inter-relationship formulated into a piecewise linear model in section 4. An analysis of the parameters of EEO policy and their effect on promotion possibilities is then provided in section 6. Finally, the numerical example in section 7 further clarifies the relationship between EEO policy and the social structure of the organization.

It is our intention in this paper to lay the groundwork for analyzing the promotion opportunities of a heterogeneous work force population. Therefore, the bulk of this paper is devoted to an expository discussion of the definitions and conditions arising from EEO and Affirmative Action conditions. Subsequent papers will be devoted to further extensions and applications to personnel planning problems.

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[2] See also Charnes, Cooper and Niehaus $\underline{/}$ 3 $\underline{/}$ and Lewis $\underline{/}$ 9 $\underline{/}$.

2. Conceptual Background

For a given organization we distinguish between two structures: (a) the **organizational structure** reflecting the hierarchy of grades and the allowable mobility patterns within it; and (b) the **social structure** providing a heterogeneous portrayal of the attributes of the population, k in number, (e.g., sex, ethnicity, educational level, seniority, skill or occupation, etc.).

We direct our analysis in this paper to a single origin grade, say i, of an organizational (graded) structure and a single destination grade, j. We assume that mobility from grade i to grade j is attribute-specific, and that a functional relationship exists between an individual's attributes and his probability of promotion. For example, an increase in an individual's job seriority and/or educational level will tend to bring about a concurrent increase in his promotion probability.

The population in each grade is assumed to be **heterogeneous**, classified by its attribute combinations. We use attribute combinations of individuals to obtain n mutually exclusive homogeneous sub-populations (i.e., groups) of individuals. Two individuals who are alike in every attribute but one, say sex, will belong to two distinct groups.

To formalize the above, we let

x_α^ℓ = an index measurement of the α attribute associated with population group ℓ, where $\ell = 1,\ldots,n$ and $\alpha = 1,\ldots,k$, so that we may associate with each homogeneous group ℓ an attribute vector, x^ℓ.

Further, we let

$p_{ij}(x^\ell)$ = the promotion probability from grade i to grade j of
an individual possessing the group ℓ attribute combination.

Because in the present discussion we focus our attention on a single time period, we suppress the time notation in our formulation.

We next partition the set of attributes into two subsets: (a) invariant individual characteristics such as ethnicity and sex; and (b) those attributes that, for purposes of exposition, we will here define as variant over a continuous scale. We will use the invariant attributes for the primary purpose of further labeling and describing the various population groups. Let the number of invariant attributes be c < k. We then assume that the first derivatives of the probability function $p_{ij}(x^\ell)$ all exist and are continuous over the range of the (k-c = m) continuous attributes. Thus, we have continuous partial derivatives,

$$f_{ij\alpha}(x^\ell) = \frac{\partial p_{ij}(x^\ell)}{\partial x_\alpha^\ell} \, , \qquad \alpha = 1,\ldots,m, \tag{1}$$

denoting the marginal effect of a change in an attribute index on the associated promotion probability. We shall hereafter refer to this as the MPPE--the marginal promotion probability effect.

For any given set of invariant attributes, a proportional response function of the promotion probability to variations in the attribute index is defined as its elasticity, viz.,

$$\varepsilon_{ij\alpha}(x^\ell) = \frac{\partial \log p_{ij}(x^\ell)}{\partial \log x_\alpha^\ell} = \frac{f_{ij\alpha}(x^\ell) \, x_\alpha^\ell}{p_{ij}(x^\ell)} \, , \qquad \alpha = 1,\ldots,m. \tag{2}$$

We also define

$$h_{ij}(x^\ell) = \sum_{\alpha=1}^{m} \varepsilon_{ij\alpha}(x^\ell) = \sum_{\alpha=1}^{m} \frac{f_{ij\alpha}(x^\ell)}{p_{ij}(x^\ell)} x_\alpha^\ell, \tag{3}$$

which provides a local measure of change in the promotion probability as a result of a scale change in the attribute index.[3]

We next observe that

$$\begin{aligned} h_{ij}(x^\ell) &= \sum_{\alpha=1}^{m} \varepsilon_{ij}(x^\ell) \\ &= \sum_{\alpha=1}^{m} \frac{f_{ij\alpha}(x^\ell)\, x_\alpha^\ell}{p_{ij}(x^\ell)} \\ &= \frac{1}{p_{ij}(x^\ell)} \sum_{\alpha=1}^{m} f_{ij\alpha}(x^\ell)\, x_\alpha^\ell, \end{aligned} \tag{4}$$

or,

$$p_{ij}(x^\ell) = \frac{1}{h_{ij}(x^\ell)} \sum_{\alpha=1}^{m} f_{ij\alpha}(x^\ell)\, x_\alpha^\ell. \tag{5}$$

Equation (5) is a definition in which $h_{ij}(x^\ell)$ is a function of the vector x^ℓ. If $h_{ij}(x^\ell)$ is a constant, then equation (5) conforms to Euler's Theorem. In any case, it enables us to express each promotion probability as a function of the promotion elasticities, $\varepsilon_{ij\alpha}$, and the change in these probabilities associated with alterations in each attribute x_α^ℓ, aggregated over the set of m variant attributes.

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[3] These definitions are consistent with the economic nomenclature of "output elasticity" and "elasticity of production." Cf., e.g., Intriligator $\underline{/\ 7\ \underline{/}}$, pp. 181-182.

3. Equal Employment Opportunity Considerations

We now turn our attention to incorporating manpower policy and relevant considerations of equity in our model design. This is accomplished by introducing a principle of equal employment opportunity, (EEO), that provides for a way to distinguish between the effect on promotion opportunities of an individual attribute, and the population group to which it belongs. Allowances are then made to account for the possibility of discretionary decisions by managers.

Initially, we observe that while the combination of attributes in the vector x^ℓ is generally determined by individual characteristics,[4] the marginal promotion probability effect (MPPE) values in equation (1) generally contain some degree of managerial discretion. The latter is manifested in various ways such as the formulation and approval of job specifications at a destination grade j, and related organization-wide policies and objectives. For example, if promotion to j is associated with movement to a higher level administrative job, then we would expect $f_{ij\alpha}(x^\ell)$ to increase for any specified x_α^ℓ, when the index α is associated with attributes indicative of administrative skills. Of course, when it is not associated with administrative skills, the sign of the related MPPE is not decisively determined.

For each population group we define a distinct x^ℓ with an associated $f_{ij\alpha}(x^\ell)$. Any variation in x^ℓ would generally imply a fluctuation in the $f_{ij\alpha}(x^\ell)$. Consider, for example, two individuals who are alike in

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[4]Cf., the system for setting executive compensation in Charnes and Cooper $\underline{/\ 1\_/}$, Chapter X, where managerial incentives are provided for altering these individual characteristics. See also Tinbergen $\underline{/\ 11\_/}$ and $\underline{/\ 12\_/}$ for a job selection model based on matching individual attributes and job characteristics, and Wise $\underline{/\ 13\_/}$ for a related individual choice model.

every attribute except sex. A clear case of discrimination would then
be present if each incremental year of seniority was weighted differently
in promotion considerations.

We may formalize what is involved as follows. Consider the
attribute vector for a pair of individuals in any two different popula-
tion groups, say x^U and x^ℓ, respectively. Discrimination will be present
if we have $x^U = x^\ell$, but,

$$f_{ij\alpha}(x^\ell) \neq f_{ij\alpha}(x^U), \qquad \alpha = 1,\ldots,m. \qquad (6.1)$$

Furthermore, if only the αth attribute is pertinent, then we may drop
the requirement, $x^U = x^\ell$, and still say that discrimination will be
present if equation (6.1) holds with $x^U_\alpha = x^\ell_\alpha$. In either case, the
presence of discrimination is detected by reference to alterations
in the corresponding MPPE, which may be attribute-specific or not.

Note that we do not require equal promotion probabilities, as would
be the case if we were trying to equalize all opportunities, independent
of past considerations. This concept might well be illustrated by the
following example. Consider a minority employee with 5 years of
experience and 2 years of education, and a non-minority employee with
5 years of experience and 7 years of education. Assume that these are
the only attributes to be considered. If both employees increase their
years of experience by one, the marginal effect of this change on their
respective probabilities of promotion should be the same, in the absence
of discrimination, although their respective probabilities of promotion
may be different resulting from different levels of educational attain-
ment. Note that the marginal effect of the increase in experience is

required by this principle to be equal for both cases, even though one
employee is of minority status and the other is not.

In the above example, the years of experience were the same for both
groups, and so, evidently, equal employment opportunity requires the
promotion probabilities to be incremented by the same degree for an equal
change in experience. But now, assume that the relevant gradations for
education are 1-4 years, 5-6 years and 7-10 years. If both employees
increase their years of education by one, the fact that they started with
different amounts of 2 and 7 years, respectively, will result in a dif-
ferent incremental change to their respective promotion probabilities.
However, in the event that the MPPE values associated with these one year
educational increments are the same, it need not be implied that the EEO
principle follows. We would, however, like to alter equation (6.1) to
obtain

$$f^{\ell}_{ij\alpha} = f^{\upsilon}_{ij\alpha}, \qquad \alpha = 1,\ldots,m, \text{ and}$$
$$\ell, \upsilon = 1,\ldots,n, \tag{6.2}$$

where

$$f^{\ell}_{ij\alpha} \equiv f_{ij\alpha}(x^{\ell}_1,\ldots,x^{\ell}_{\alpha},\ldots,x^{\ell}_m),$$

which is constant for a given x^{ℓ}_{α} and all x^{ℓ}_{γ}, $\gamma \neq \alpha$, but varies with
x^{ℓ}_{α} alone. Thus, the definition in equation (6.2) implies the presence
of an EEO policy.

The fact that the total probability of promotion is left unattended
at this point in order to focus on the marginal changes is also deliberate.

We want to reserve the total promotion for consideration in terms of affirmative action, and subsequently examine how these EEO and affirmative action characterizations interact in a total organizational personnel program.

The above characterization of equal employment opportunity provides a vehicle for transforming equation (5) into the following form:

$$p_{ij}(x^{\ell}) = \frac{1}{h_{ij}(x^{\ell})} \sum_{\alpha=k+1}^{m} f_{ij\alpha} \, x_{\alpha}^{\ell}, \tag{7}$$

for any of the given invariant attributes $\{x_{\alpha}^{\ell}, \ \alpha = 1,\ldots,k\}$.

4. Model Definition

To help clarify the intended impact of the preceding developments, we now proceed with an explicit piecewise linear approximation of the functional form expressed in equation (6.2). This will entail one too many spaces partitioning the variant attribute indices into level gradations. For example, in the case of educational attainment, the continuum would be replaced by gradation into, e.g., 1-4 years, 5-6 years and 7-10 years. We can accomplish the above by assigning a different index α_θ to each of the applicable education classes.

For our piecewise linear approximation we write

$$f^\ell_{ij\alpha} = (x^\ell_\alpha - x^\ell_{\alpha_{N_\alpha}}) \, \beta_{\alpha_{\theta+1}} + \sum_{\theta=1}^{N_\theta} (x^\ell_{\alpha_\theta} - x^\ell_{\alpha_{\theta-1}}) \, \beta_{\alpha_\theta}, \tag{8}$$

where

$$N_\alpha = \max \theta : x^\ell_{\alpha_\theta} < x^\ell_\alpha,$$

$\{x^\ell_{\alpha_0}, x^\ell_{\alpha_1}, x^\ell_{\alpha_2}, \ldots, x^\ell_{\alpha_\theta}, \ldots, x^\ell_{\alpha_{M_\alpha}}\}$ denotes the set of predetermined levels of gradation, and β_{α_θ} denotes the slope of the linear segment between the two points $(x^\ell_{\alpha_{\theta-1}}, x^\ell_{\alpha_\theta})$ on the partitioned scale of the α attribute measurement. Next, by inserting equation (8) in equation (5), we obtain a quadratic form for $p_{ij}(x^\ell)$ with piecewise linear derivatives,

$$p_{ij}(x^\ell) = \frac{1}{h_{ij}(x^\ell)} \sum_{\alpha=1}^{m} \left\{ (x^\ell_\alpha - x^\ell_{\alpha_{N_\alpha}}) \, \beta_{\theta+1} + \sum_{\theta=1}^{N_\alpha} (x^\ell_{\alpha_\theta} - x^\ell_{\alpha_{\theta-1}}) \, \beta_{\alpha_\theta} \right\} x^\ell_\alpha$$

$$= \frac{1}{h_{ij}(x^\ell)} \sum_{\alpha=1}^{m} \left\{ (x_\alpha^\ell - x_{\alpha_{N_\alpha}}^\ell) x_\alpha^\ell \beta_{\theta+1} + \sum_{\theta=1}^{N_\alpha} (x_{\alpha_\theta}^\ell - x_{\alpha_{\theta-1}}^\ell) x_\alpha^\ell \beta_{\alpha_\theta} \right\}. \quad (9)$$

In Figure 1, below, we show the curvalinear form of $f_{ij\alpha}$ approximated by a piecewise linear equation. The components of equation (9) are shown on the right-hand side, corresponding to the segments of the linear approximation.
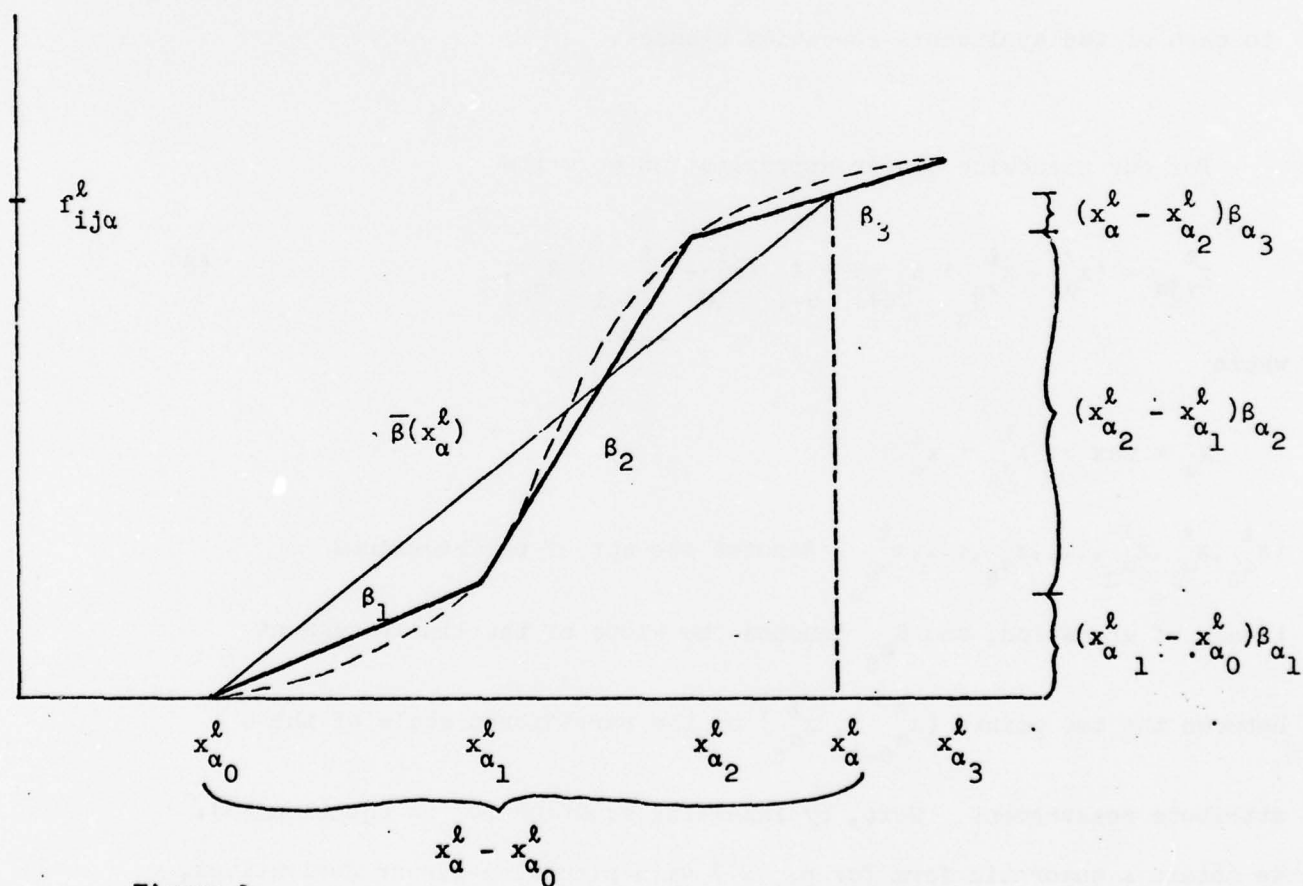

Figure 1.

We can summarize this model in matrix form by letting:

$$\beta = \left[\beta_{1_1}, \ldots, \beta_{1_{M_1}}, \ldots, \beta_{\alpha_1}, \ldots, \beta_{\alpha_{M_\alpha}}, \ldots, \beta_{m_1}, \ldots, \beta_{m_{M_m}}\right]^T, \tag{10}$$

represent the (Mx1) vector of associated EEO "weights," i.e., MPPE's, for each of the attributes over $\alpha = 1, \ldots, m$, where $M = \sum\limits_{\alpha=1}^{m} M_\alpha$;

$$X = \atop (n \times M) \left[\begin{matrix} D_1^1 x_1^1 & \cdot & \cdot & \cdot & D_\alpha^1 x_\alpha^1 & \cdot & \cdot & \cdot & D_m^1 x_m^1 \\ \cdot & & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & & \cdot \\ D_1^\ell x_1^\ell & \cdot & \cdot & \cdot & D_\alpha^\ell x_\alpha^\ell & \cdot & \cdot & \cdot & D_m^\ell x_m^\ell \\ \cdot & & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & & \cdot \\ \cdot & & & & \cdot & & & & \cdot \\ D_1^n x_1^n & \cdot & \cdot & \cdot & D_\alpha^n x_\alpha^n & \cdot & \cdot & \cdot & D_m^n x_m^n \end{matrix}\right],$$

where a typical subcomponent vector of a row in X is

$$D_\alpha^\ell = \left[(x_{\alpha_1}^\ell - x_{\alpha_0}^\ell), \ldots, (x_{\alpha_{N_\alpha}}^\ell - x_{\alpha_{N-1}}), (x_\alpha^\ell - x_{\alpha_{N_\alpha}}^\ell), 0, \ldots, 0\right], \tag{12}$$

a (1xM_α) vector with the last ($M_\alpha - N_\alpha$) components equal to zero. Thus, in the above example, for a person with 5 years of education, (e.g., a completed Beccalaureate degree and 1 year toward completion of a Masters degree), $D_\alpha^\ell = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix}$, while for a person with 7 years of education, $D_\alpha^k = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix}$.

For further facility in interpretation, we can rewrite equation (12) as

$$D_\alpha^\ell = (x_\alpha^\ell - x_{\alpha_0}^\ell)\, d_\alpha^\ell, \tag{13}$$

where d_α^ℓ denotes the vector described by the relative distribution of the α attribute-gradation specific to population group ℓ, with components

$$d_{\alpha_\theta}^\ell = \frac{(x_{\alpha_\theta}^\ell - x_{\alpha_{\theta-1}}^\ell)}{x_\alpha^\ell - x_{\alpha_0}^\ell} \geq 0, \text{ and } \sum_{\theta=1}^{N_\alpha} d_{\alpha_\theta}^\ell = 1. \tag{14}$$

Thus, in the above example, for a person with 5 years of education, $d_\alpha^\ell = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & 0 \end{bmatrix}$, and for a person with 7 years of education, $d_\alpha^k = \begin{bmatrix} \frac{4}{7} & \frac{2}{7} & \frac{1}{7} \end{bmatrix}$. Hence,

$$f_{ij\alpha}(x^\ell) = (x_\alpha^\ell - x_{\alpha_0}^\ell) \sum_{\theta=1}^{M_\alpha} d_{\alpha_\theta}^\ell \beta_{\alpha_\theta} = (x_\alpha^\ell - x_{\alpha_0}^\ell)\, \bar{\beta}(x_\alpha^\ell), \tag{15}$$

where

$$\bar{\beta}(x_\alpha^\ell) = \sum_{\theta=1}^{M_\alpha} d_{\alpha_\theta}^\ell \beta_{\alpha_\theta} \tag{16}$$

is the weighted mean of the predetermined weights of the gradations. Note that $d_{\alpha_\theta}^\ell$ is a piecewise linear function of x_α^ℓ. Again, in the above example, for an individual with 5 years of education, $\bar{\beta}(x_\alpha^\ell) = (\frac{4}{5})\beta_{\alpha_1} + (\frac{1}{5})\beta_{\alpha_2} + (0)\beta_{\alpha_3}$ and for one with 7 years of education, $\bar{\beta}(x_\alpha^k) = (\frac{4}{7})\beta_{\alpha_1} + (\frac{2}{7})\beta_{\alpha_2} + (\frac{1}{7})\beta_{\alpha_3}$.

To sum up, X is the (nxM) matrix of attribute measures associated with the population groups corresponding to the piecewise linear structure of the model;

$$P_{(n\times1)} = \left[p_{ij}(x^\ell), p_{ij}(x^2), \ldots, p_{ij}(x^n)\right]^T$$

denotes the (nx1) vector of the set of population group-specific promotion probabilities over the $\ell = 1, \ldots, n$ population groups; and $\hat{h}$ designates an (nxn) diagonal matrix constructed from the reciprocals of the "total elasticities," $h_{ij}(x^\ell)$, as defined in equation (3). Using the above definitions, we can express equation (7) as

$$P = \hat{h}X\beta. \tag{17}$$

5. Affirmative Action Interpretations

Let W_ℓ denote the number of members of group ℓ in grade i. Then, $W_\ell p_{ij}(x^\ell)$ represents the expected number of people of group ℓ promoted to grade j. Thus, the aggregate promotion rate is given by the ratio

$$p_{ij} = \sum_{\ell=1}^{n} \frac{W_\ell p_{ij}(x^\ell)}{\sum\limits_{\ell=1}^{n} W_\ell} = \sum_{\ell=1}^{n} \omega_\ell p_{ij}(x^\ell), \tag{18}$$

where $\omega_\ell = W_\ell / \sum\limits_{\ell=1}^{n} W_\ell$ is the proportion of the total population that belongs to group ℓ. Denoting $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, from equations (17) and (18), we obtain

$$p_{ij} = \omega P. \tag{19}$$

This further suggests that the scale of the diagonal elements of $\hat{h}$ must be selected to satisfy $1 \geq p_{ij}(x^\ell) \geq 0$, and $1 \geq p_{ij} = \omega P \geq 0$.

Equations (17) and (19) provide a linear mapping from a set of attribute-specific weights, β, to the vector of population group-specific promotion rates, P, by means of an affirmatively weighted social structure, $\hat{h}X$. Thus, one problem might be to delineate a discretionary range for β and/or $\hat{h}$. This can provide initial guidance for managerial consideration of equal employment (and also affirmative action) as a part of a comprehensive personnel program. The way this may be done will be illustrated after we first develop the relevant affirmative action parameters in this section.

We now observe that $\omega\hat{h} = \hat{h}\omega$, hence from equation (19),

$$p_{ij} = \omega P$$
$$= \omega \hat{h} X \beta$$
$$= h \hat{\omega} X \beta \qquad\qquad (20)$$
$$= hA\beta,$$

where $A = \hat{\omega}X$, such that the social structure matrix, X, is weighted by
the level of participation of each population group in the grade, h, is a
($1 \times n$) row vector with $1/h_{ij}(x^\ell)$ for its components, $\hat{\omega} = \text{diag }(\omega)$ and
$\hat{h}\omega = \hat{\omega}h$. The elements of the vector h constitute the affirmative action
components we wish to consider next.

Note first that, for a given β, reflecting job-related, attribute-
specific parameters, the social structure, $A = \hat{\omega}X$ dictates a particular
range of choices for affirmative action weights in order that the bilinear
form $hA\beta$ will have a value of p_{ij} for all discretionary sets, $\{\beta, h\}$.
This is necessary because we want to characterize affirmative action in
terms of individual population groups' probabilities for promotion,
$p_{ij}(x^\ell)$.

It may be recalled that in section 2 of this paper we presented
the term $h_{ij}(x^\ell)$ to denote the total elasticity (i.e., the degree of
homogeneity in the case of a homogeneous function) of a promotion proba-
bility function $p_{ij}(x^\ell)$. The total elasticity was defined as a sum
of the partial elasticities of a promotion probability function with

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[6] A game theoretic interpretation of equation (20) is discussed in
a subsequent paper, K.A. Lewis and A.P. Schinnar, "A Game of Quotas
for Equal Opportunity Employers," (in preparation).

respect to changes in its attribute indices. The components of the discretionary row vector h in equation (20), $1/h_{ij}(x^\ell)$, are the "mathematical" reciprocals of the respective total elasticities. The operational implication of this connection with respect to interpretations related to affirmative action, is, therefore, the following: population groups normally possessing attribute structures that demonstrate low elasticities, would require a high degree of "affirmative action" input--i.e., a small $h_{ij}(x^\ell)$ implies a large $1/h_{ij}(x^\ell)$. In other words, a large "affirmative action" effect is required to compensate for an otherwise historically low tendency to be promoted.

Note from equation (5) that these $1/h_{ij}(x^\ell)$ values are "weights" used in the definition of $p_{ij}(x^\ell)$. Thus, we require a "heavy" weight to represent an affirmative action policy. Similarly, a low discretionary weighting placed on some population-specific attribute combination implies a high total elasticity in the associated promotion probability function. This, in turn, implies a low elasticity reflected in the functional responses to changes in the population attribute index scores. This conceptual connection between the elasticity of a population group's promotion probability function and its affirmative action policy is inherently, as well as operationally, consistent. We accord $1/h_{ij}(x^\ell)$ a discretionary interpretation for use by management in order to include differential weighting on the various population groups. Thus, well-defined affirmative action strategies can be incorporated into the model development and their respective impacts clearly noted.

6. Analysis of Promotion Possibilities

We turn now to illustrate the explicit relationship between the Affirmative
Action and Equal Employment Opportunity parameters by examining the range
of promotion possibilities for various population groups. We use here an
analog from production economics, especially Lancaster's formulation $\underline{/\,8\,\underline{/}}$
of a production possibility space, constrained by a budget space. This
approach has been made operational and wholly computational in $\underline{/\,5\,\underline{/}}$, which
we follow here for the calculation of efficient trade-offs, i.e., substitu-
tions and complementarities, among the promotion opportunities of various
population groups.

For a given h, ω, X and p_{ij}, the relationship specified in equation (17)
describes a transformation, or mapping, from a space of aggregate promotion
probabilities for β into a space of individual promotion probabilities with
points $p_{ij}(x^{\ell})$. The transformation is submitted to a constraint, as in
equation (20) as follows. Formally we have an aggregate promotion proba-
bility set defined as

$$G \equiv \{\beta: \; [\omega \hat{h} X]\beta = p_{ij}, \; \beta \geq 0\}. \tag{21}$$

Note that without loss of generality we can let $\beta \geq 0$ by adjusting the
sign of the respective columns in X. The image in the space of individual
promotion probabilities for the $\hat{h}X\beta$ transformation is the set

$$V \equiv \{p_{ij}(x^{\ell}): \; P = [\hat{h}X]\beta, \; [\omega \hat{h}X]\beta = p_{ij}, \; \beta \geq 0\}. \tag{22}$$

Our analysis in this section will focus on the properties of V and
its relationship to the affirmatively weighted social structure matrix
in each grade, $\hat{h}X$, and its aggregate form, $\omega \hat{h}X$, which will be viewed here

as the analog of prices in $\underline{\int} 8 \underline{\int}$, and p_{ij}. The properties of V are as follows[7]. The aggregate promotion probability in G is the set of all convex combinations of the following extreme points,

$$
\begin{bmatrix} p_{ij}/\omega\hat{h}X_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\begin{bmatrix} 0 \\ p_{ij}/\omega\hat{h}X_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\cdots
\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ p_{ij}/\omega\hat{h}X_M \end{bmatrix}
$$

where X_s denotes column s in X, $s = 1,\ldots,M$. V is the image set of G that consists of all convex combinations of the images of the extreme points above. While every extreme point of V is the image of an extreme point in G, an extreme point of G is not necessarily an extreme point of V. Hence, the polytope formed by the above extreme points can be described in terms of the convex hull defined about the points

$$
\left(\frac{p_{ij}}{\omega\hat{h}X_s}\right)\hat{h}X_s. \tag{23}
$$

Assuming the existence of a differentiable untility function, $U(P)$, defined over the set of individual promotion probabilities, and that $\partial U(P)/\partial p_{ij}(x^\ell) > 0$, for all ℓ, we can define the "efficient" promotion frontier of V in terms of the problem

---

[7] See $\underline{\int} 5 \underline{\int}$ and $\underline{\int} 8 \underline{\int}$.

$$\underset{P}{\text{maximize}} \; U(P), \qquad \text{subject to } P \leqslant V, \qquad (24)$$

where an optimum point, P^*, is necessarily a boundary point. We further define the efficient set of promotion probabilities,

$$\{\beta: \; [\hat{h}X]\beta \leqslant V^*, \; [\omega\hat{h}X]\beta = p_{ij}, \; \beta \geq 0\}, \qquad (25)$$

where V^* denotes the efficiency frontier of promotion possibilities such that $P^* \leqslant V^*$.

As shown in $\underline{/\,8\,\underline{/}}$, once P^* is known, β can be obtained from the ordinary linear program

$$\underset{\beta}{\text{minimize}} \; [\omega\hat{h}X]\beta, \; \text{subject to } [\hat{h}X]\beta = P^*, \; \beta \geq 0, \; P^* \leqslant V^*, \qquad (26)$$

and the associated dual program

$$\underset{\pi}{\text{maximize}} \; \pi P^*, \; \text{subject to } \pi[\hat{h}X] \leq \omega\hat{h}X, \qquad (27)$$

where the components of π are otherwise unrestricted. At an optimum point, if it exists, we have

$$p_{ij} = \omega\hat{h}X\beta^* = \omega P^* = \pi^* P^*, \qquad (28)$$

where π^*, $\ell = 1,\ldots,n$, are the shadow prices associated with the promotion of population group ℓ.

The unrestricted signs of the shadow prices, π^*, customarily reflect the sensitivity of the aggregate promotion probability, p_{ij}, to marginal variations in $\{p_{ij}^*(x^\ell)\}$. Since we want to consider adjustments in P^* such that $P^* \leqslant V^*$ for which $p_{ij} = \omega P$ --i.e., stay on the surface of the efficient

promotion frontier--we may also accord a trade-offs interpretation for the sign and scale of π_ℓ^*.

Now, consider a change in the $p_{ij}(x^\ell)$ of two population groups. As shown in $\underline{/\,5\,\underline{/}}$, if $\pi_\ell^* = 0$ for at least one, the substitutability does not occur. If the signs of π_ℓ^* are the same for both, we have a case in which the $p_{ij}(x^\ell)$ of the two are <u>substitutes</u>. If the sign of π_ℓ^* is different, complementarity holds between the two groups and substitution, again, does not occur. When substitution occurs, the ratio between the respective shadow prices reflects the trade-off rate between the promotion opportunities of the two population groups.

The only remaining difficulty with implementing the above model is access to the utility function $U(P)$. We adopt here the "goal focusing" method, developed in $\underline{/\,5\,\underline{/}}$, where the utility function is replaced by a functional of goal artifact deviations:

$$\text{minimize } -\eta p_{ij} + \sum_{\ell k}(\sigma_{\ell k}^+ \delta_{\ell k}^+ + \sigma_{\ell k}^- \delta_{\ell k}^-) \tag{29}$$

$$\text{subject to } p_{ij}(x^\ell) - \rho_{\ell k}p_{ij}(x^k) - \delta_{\ell k}^+ + \delta_{\ell k}^- = 0; \quad \ell, k = 1, \ldots, n;$$
$$\ell \neq k.$$

$$\omega P = p_{ij}$$

$$[\hat{n}X]\beta = P$$

$$\underline{0} \leq \bar{c} \leq P \leq \underline{c} < \underline{1}$$

$$R\beta \leq 0$$

$$P, \beta, \delta_{\ell k}^+, \delta_{\ell k}^- \geq 0,$$

where $\eta = \sum\limits_{\ell k}(\sigma^+_{\ell k}\delta^+_{\ell k})$, $\sigma^+_{\ell k}$, $\sigma^-_{\ell k}$ are non-negative weights associated with positive and negative deviations, $\delta^+_{\ell k}$, $\delta^+_{\ell k} \geq 0$, from the goal artifacts in the constraint, and $\rho_{\ell k}$ is a scalar ℓk^{th} constraint reflecting the desired or existing relationship between $p_{ij}(x^\ell)$ and $p_{ij}(x^k)$. Thus, for example, $\rho_{\ell k} = 1$ implies a desired equality of individual promotion probabilities for population groups ℓ and k. $\underline{c}$ and $\bar{c}$ are corresponding vectors of upper and lower bounds on promotion probabilities, respectively. η is a large positive scalar, so that minimizing $-\eta p_{ij}$ and, hence, maximizing ηp_{ij}, will ensure the attainment of the efficiency frontier for P^*. The final constraint set, $R\beta \leq 0$, designates desired relationships between the β parameters, such as, $\beta_1 \geq \beta_2$, that reflect job-related characteristics at the destination grade.

In the above approach, we want to be "as close as possible" to all goals, but with $p_{ij}(x^P)$ also at the highest possible level. Thus, bearing in mind that generally there are many efficient points in V^* and that a utility function focuses our attention on only a few of them, using program (29) we have performed a similar focusing via the $\rho_{\ell k}$ goal artifacts.

The goal programming forum of the above problem is particularly useful when the constraints on P and β forestall the attainment of a particularly desired P^*, such as having P^* reflect equal promotion probabilities for all population groups involved. That is, when a set of equal opportunity goals are inconsistent with other organizational and social goals and constraints and cannot be attained simultaneously, the goal programming approach directs us to alternatives

that come "as close as possible" to achieving equal opportunities in accord with other pending organizational considerations.

If the set of goal artifacts, $\rho_{\ell k}$, is fully specified for all ℓ and k, and is wholly consistent with the remaining constraints in program (29), we can simplify the program as follows. From the set

$$p_{ij}(x^\ell) - \rho_{\ell k}p_{ij}(x^k) = 0, \qquad \ell, k = 1,\ldots,n, \qquad (30)$$

we have (n-1) linearly independent equations, which, when coupled with $\omega P = p_{ij}$, gives the non-singular system

$$\begin{bmatrix} 1 & -\rho_{12} & & & \\ 1 & & -\rho_{13} & & \\ \vdots & & & \ddots & \\ 1 & & & & -\rho_{1n} \\ \omega_1 & \omega_2 & \omega_3 & \cdots & \omega_n \end{bmatrix} \begin{bmatrix} p_{ij}(x^1) \\ p_{ij}(x^2) \\ \vdots \\ p_{ij}(x^n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ p_{ij} \end{bmatrix}, \qquad (31)$$

whose solution is

$$p_{ij}^*(x^\ell) = \frac{p_{ij}}{\rho_{i\ell}} \left(\omega_1 + \frac{\omega_2}{\rho_{12}} + \frac{\omega_3}{\rho_{13}} + \ldots + \frac{\omega_n}{\rho_{1n}}\right)^{-1}, \quad \ell = 1,\ldots,n. \qquad (32)$$

Note that for equal promotion probabilities $\rho_{i\ell} = 1$ implies that $p_{ij}(x^\ell) = p_{ij}$ for all ℓ. Now that P^* is available, we can apply program (26) directly.

7. Numerical Example

In this section we provide a simple example in order to illustrate the workings of the concepts outlined in the previous sections. Specifically, we focus on the trade-offs among the promotion opportunities of various population groups with reference to Equal Employment Opportunity and Affirmative Action considerations. Consider the mobility between two grades in an organization. The pool of populations consists of two social groups whose attribute data is represented in the following table:

Population Group	Pop. Size	Attributes				Years of related experience to job j.
		Education		Seniority in job i.		
		Undergrad.	Grad.	$\leq$ 3 yrs.	$>$ 3 yrs.	
1. Male staff	6	4	0	3	0	1
2. Female staff	4	4	0	3	2	0
		β_U	β_G	β_{S1}	β_{S2}	β_E

The first row describes the group of male staff with undergraduate educations, less than 3 years of seniority in the present grade and one year of related experience to the anticipated range of tasks in job j. The second row consists of female staff holding undergraduate degrees and exhibiting high seniority in the present grade. This higher seniority level is representative of a bottleneck effect, possibly resulting from past discrimination in promotions. The β parameters at the bottom of the table denote the marginal effect of variation in the attribute structure of each population group in the following manner. E.g., S2 provides the

marginal effect of additional seniority for the female staff. Of course, the gradation of education and seniority can be further refined in the above example. Also, related experience can be accompanied by scores on performance and other qualifications.

The matrix X of attribute information can be constructed according to equation (11) by multiplying each score in the respective gradations of an attribute with the total interval x_α^ℓ, thus

$$X = \begin{bmatrix} 4(4) & 0(4) & 3(3) & 0(3) & 1(1) \\ 4(4) & 0(4) & 3(5) & 2(5) & 0(0) \end{bmatrix} = \begin{bmatrix} 16 & 0 & 9 & 0 & 1 \\ 16 & 0 & 15 & 10 & 0 \end{bmatrix}.$$

Let $h = \begin{bmatrix} .1 & .2 \end{bmatrix}$, reflecting a higher affirmative action weight for the promotion of female personnel. Hence,

$$\hat{h}X = \begin{bmatrix} .1 & 0 \\ 0 & .2 \end{bmatrix} \begin{bmatrix} 16 & 0 & 9 & 0 & 1 \\ 16 & 0 & 15 & 10 & 0 \end{bmatrix} = \begin{bmatrix} 1.6 & 0 & .9 & 0 & .1 \\ 3.2 & 0 & 3 & 2 & 0 \end{bmatrix}$$

provides the transformation matrix between the aggregate and individual promotion spaces. As we have 6 men and 4 women in the grade, $\omega = \begin{bmatrix} .6 & .4 \end{bmatrix}$, and the aggregate promotion equation becomes

$$p_{ij} = \omega\hat{h}X\beta$$

$$= (2.24)\beta_U + (0)\beta_G + (1.74)\beta_{S1} + (.8)\beta_{S2} + (.06)\beta_E.$$

Then, the extreme points in G are

$$\begin{bmatrix} p_{ij}/2.24 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ p_{ij}/1.74 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ p_{ij}/.8 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ p_{ij}/.06 \end{bmatrix}.$$

The image of these in V is

$$
\overset{\text{(a)}}{\begin{bmatrix} .71p_{ij} \\ 1.42p_{ij} \end{bmatrix}} \overset{\text{(b)}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \overset{\text{(c)}}{\begin{bmatrix} .52p_{ij} \\ 1.72p_{ij} \end{bmatrix}} \overset{\text{(d)}}{\begin{bmatrix} 0 \\ 2.5p_{ij} \end{bmatrix}} \overset{\text{(e)}}{\begin{bmatrix} 1.67p_{ij} \\ 0 \end{bmatrix}} ,
$$

which gives rise to the convex polyhedral set portrayed in Figure 2 by

the "shaded" region, for p_{ij} = .1.
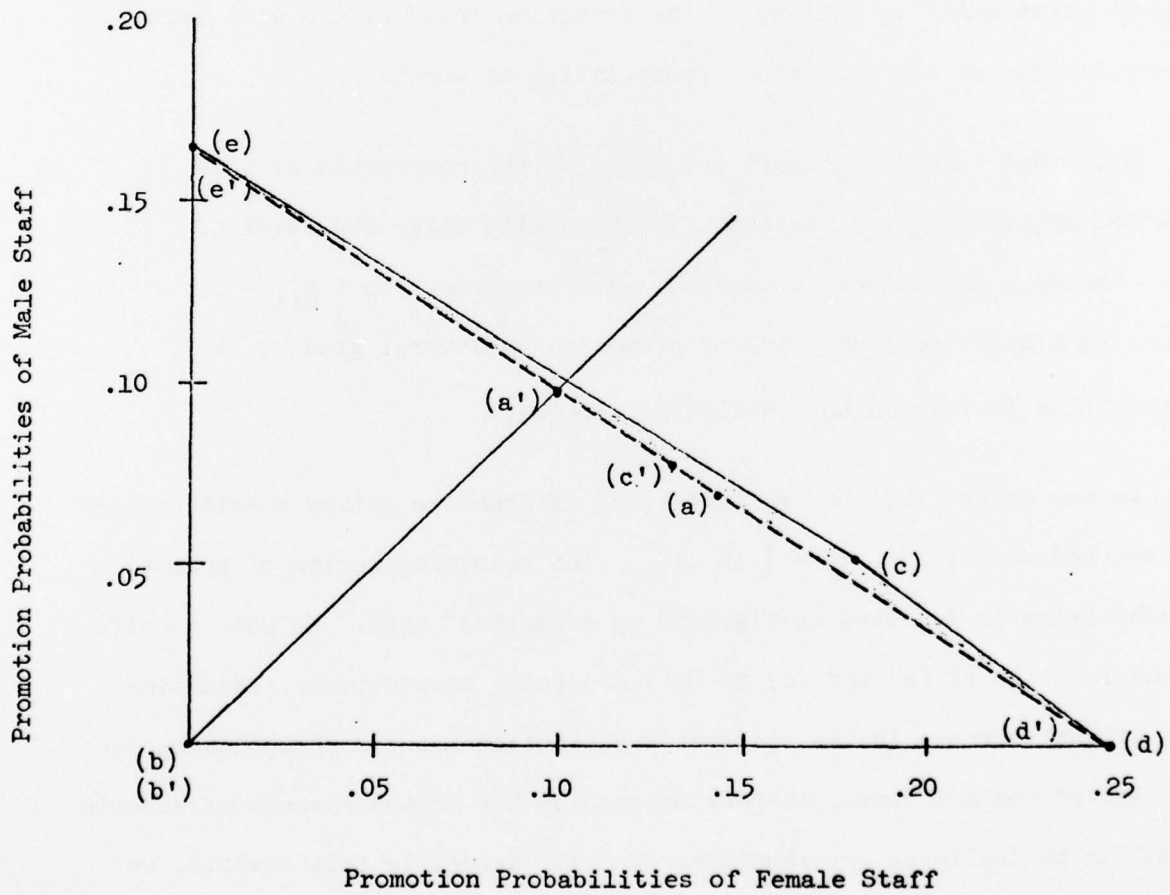
The efficiency frontier with maximum possible $p_{ij}(x^{\ell})$ is clearly portrayed by points (d), (c) and (e), implying that considerations related to seniority and previous experience will yield the highest individual promotion rates. Undergraduate education, (a), is close to the frontier, but is an interior point in the promotion possibility polytope. The ray through the origin reflects the points of equal promotion opportunities, $p_{ij}(x^1) = p_{ij}(x^2)$, and intersects the segment (e) - (c) on the efficiency frontier at the point .1, which is at the level of p_{ij}. The slope of the segment between (e) and (c) is -.67, reflecting the trade-offs between the promotion of men and women. It suggests that small variations in the promotion rates of men will have larger effects on the promotion opportunities of women.

Note that in this analysis the scale of the components of h is selected arbitrarily and variation in these will alter the level of p_{ij}. However, the latter is constrained by reference to $\sum_j p_{ij} = 1$, requiring simultaneous modeling of promotion to several grades. We discuss this further in the concluding section.

We now change the h to reflect equal affirmative action considerations for men and women, i.e., $h = [.15 \ .15]$. The resulting region of promotion possibilities is depicted in Figure 2 by a "dotted" line. We note a shift in the location of (a) and (c) to (a') and (c'), respectively, resulting in a slight increase in the slope or substitution between promotion opportunities of men and women, thereby decreasing the responsiveness of women's promotion to declining opportunities of men. Hence, in this example, we have seen how an affirmative action policy for women has increased the

sensitivity of women to opportunities vacated by men.

To illustrate a more complicated array of interactions among promotion opportunities of personnel, we introduce a third population group to the grade, consisting of 2 new recruits with graduate education, who have no tenure in the present grade, but who have one year of related experience to the destination grade. We chose not to favor the promotion of these recruits over male personnel, but we introduce affirmative action considerations for women as before. The social structure of the present composition of the grade is

$$X = \begin{bmatrix} 4(4) & 0 & 3(3) & 0 & 1(1) \\ 4(4) & 0 & 3(5) & 2(5) & 0 \\ 4(6) & 2(6) & 0 & 0 & 1(1) \end{bmatrix} = \begin{bmatrix} 16 & 0 & 9 & 0 & 1 \\ 16 & 0 & 15 & 10 & 0 \\ 24 & 12 & 0 & 0 & 1 \end{bmatrix}.$$

The third row of X represents the newly recruited staff in this grade. We assume that these replace two quits, a man and a woman. Hence, $\omega = \begin{bmatrix} .5 & .3 & .2 \end{bmatrix}$, $h = \begin{bmatrix} .1 & .2 & .1 \end{bmatrix}$,

$$\hat{h}X = \begin{bmatrix} 1.6 & 0 & .9 & 0 & .1 \\ 3.2 & 0 & 3 & 2 & 0 \\ 2.4 & 1.2 & 0 & 0 & .1 \end{bmatrix}$$

and

$$p_{ij} = \omega \hat{h}X = (2.24)\beta_U + (.24)\beta_G + (1.35)\beta_{S1} + (.6)\beta_{S2} + (.07)\beta_E.$$

We now apply the procedure outlined at the end of section 6 to compute the trade-offs among the promotion probabilities of male and female personnel and new recruits. The results are

$$\beta_U^* = \beta_G^* = \beta_{S1}^* = 0, \qquad \beta_{S2}^* = .05, \qquad \beta_E^* = 1.0$$

$$\pi_1^*(\text{males}) = .5, \qquad \pi_2^*(\text{females}) = .3, \qquad \pi_3^*(\text{recruits}) = .2.$$

In this example, with equal promotion probabilities for all population groups, i.e., $p_{ij}(x^{\ell}) = p_{ij} = .1$, promotion is based on seniority and related experience to the destination position, with a higher marginal effect attributed to experience related to job j than is related to seniority in job i. The shadow prices, π^{*}, we found to correspond to the share of each population group in the promotion pool, i.e., $\pi^{*} = \omega$.

8. Conclusion

The procedure and relationships outlined in the preceding sections
couple the concepts of modern personnel management, in terms of the needs
associated with legal compliance, and their operational demands associated
with implementation. Organizational response to the attribute structure
of the workforce may now be considered, first, as in compliance with
Equal Employment Opportunity law, and second, with respect to the explicit
components of affirmative action policies and their individual effects
on promotion opportunities of staff.

By examining the workforce attribute structure, the decision-maker
may choose the values for the various β's, while maintaining the various
differentiations dictated by job-related requirements. Affirmative
action policy and its operational impact may then be defined through
the vector h. Such delineation of manpower policy provides an organization
with an operational framework for exploring the range of promotion stra-
tegies in compliance with Equal Employment Opportunity law, and develops
the framework for further investigation on the stance assumed by the
organization with respect to external demographic conditions.

For organization-wide planning purposes, the information supplied
by the $p_{ij}(x^{\ell})$ values can be aggregated into a Markov matrix of organiza-
tional mobility, as shown in equation (18). This will provide for a way
by which to design goal-oriented transition matrices. This, then,
represents a departure from the solely historical approach to analysis.
As we observed in the numerical example of section 7, when the Affirmative

Action program changes or the social structure, as reflected in ω, is altered due to promotion and recruitment reforms, the β parameters will change, leading to adjustments in the $p_{ij}(x^\ell)$. Thus, the Markov chain resulting from equation (18) will be non-stationary.

The approach outlined in section 6 can be used to compute the individual promotion probabilities as well as the aggregate (Markov) rate proposed above by effecting the condition that $\sum\limits_{j} p_{ij} = 1$ for each row of the aggregate transition matrix. Thus, from equation (20) we have

$$1 = \sum_{j} h_{j}\left[\hat{\omega}X\right]\beta_{j},\tag{32}$$

with an associated set of transformation matrices $h_{j}\left[\hat{\omega}X\right]\beta_{j} = p_{ij}(x^\ell)$, for use in the goal focusing program (29). Here, h_{j} will be scaled in accordance with (32). We explore this further in a sequel extension of this paper. Further extensions of this work may include the application of a scaling method for estimating Equal Employment Opportunity and Affirmative Action policy parameters, as well as game characterizations of the relationship between them.

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13. ABSTRACT

This paper develops an organizational promotion model, based on a principle of equal employment opportunity, to effect linkages between promotion opportunities and organizational responses to legal compliance. The model considers individual attributes and the manner in which they are "weighted" as contributions to human resource valuation, and, hence, to the probabilities of promotion within a given career ladder. The connection of such human resource contributions to equal employment opportunity issues provides an insight into the development of a management tool for the discretionary weighting of promotion factors for future policy-making.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Organizational Design Models (mathematical)						
Personnel Planning						
Manpower Management						
Equal Employment Opportunity						
Affirmative Action						
Manpower Mobility						